Directional regularity: Achieving faster rates of convergence in multivariate functional data

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Abstract

We introduce a new notion of regularity, called *directional regularity*, which is relevant for a wide range of applications involving multivariate functional data. We show that for anisotropic functional data, faster rates of convergence can be obtained by adapting to its directional regularity through a change of basis. An algorithm is constructed for the estimation and identification of the directional regularity for a large class of stochastic processes, made possible due to the unique replication nature of functional data. Accompanying non-asymptotic theoretical guarantees are provided. A novel simulation algorithm, which is of independent interest, is designed to evaluate the numerical accuracy of our directional regularity algorithm. Simulation results demonstrate the good finite sample properties of our estimator, which is freely available in the **R** package `direg`.