

Multivariate functional ICA for spatially indexed data*

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Abstract

We consider the case of multivariate functional data (i.e., data in the form of a continuous vector function on a given interval) that are realizations of a spatially indexed random functional variable. These data are modeled in two levels of hierarchy: the first corresponds to independent and identically distributed realizations of the functional variable at each spatial location, and the second encompasses the spatial locations that constitute the main dependent structure generating the multivariate functional data. A functional independent component analysis (ICA) approach is proposed to study these data in high-dimensional settings. We show that functional ICA provides a versatile framework for anomaly detection, sparse representation, and classification. In particular, the proposed multivariate functional approach is applied to model turbulent flows of brain activity observed in electroencephalographic recordings.

Keywords: whitening operator; kurtosis operator; multivariate functional data on a heterogeneous domain

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